

# Covariant Classification Scheme of Hadrons and Chiral States

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The essential points and physical backgrounds of the covariant level-classification scheme, recently proposed by us, are reviewed: In this scheme the general composite hadrons are classified as the multiplets with the  $\tilde{U}(12)_{SF} \times O(3,1)$  symmetry, where  $\tilde{U}(12)_{SF} \supset \tilde{U}(4)_{D.S.} \otimes SU(3)_F$ , and  $\tilde{U}(4)_{D.S.}$  is a homogeneous Lorentz transformation group for Dirac spinors. This symmetry predicts the existence of new type of chiral mesons and baryons out of the conventional level classification scheme. The  $\sigma$  nonet is a typical example of chiralons to be assigned to the  $(q\bar{q})$  relativistic  $S$ -wave state.

## §1. Introduction

(*Present status of level-classification and purpose of this talk*) There exist the two contrasting, non-relativistic and relativistic, viewpoints of level-classification. The former is based on the non-relativistic quark model (NRQM) with the approximate  $LS$ -symmetry and gives a theoretical base to the PDG level-classification. The importance of the role of a relativistic symmetry, chiral symmetry, played in hadron physics is widely accepted, and it is well-known that  $\pi$  meson octet has the property as a Nambu-Goldstone boson in the case of spontaneous breaking of chiral symmetry.

Owing to the recent progress, the existence of light  $\sigma$ -meson as chiral partner of  $\pi(140)$  seems to be established<sup>1)</sup> especially through the analysis of various  $\pi\pi$ -production processes. This gives further a strong support to the relativistic viewpoint.

Thus, the hadron spectroscopy is now being confronted with a serious problem, existence of the seemingly contradictory two viewpoints, Non-relativistic and Extremely Relativistic ones.

Recently, corresponding to the above situation, we have proposed<sup>2), 3)</sup> a covariant level-classification scheme of light-through-heavy quarks (and possibly of gluonic) hadrons, unifying the above two view-points. There we have pointed out a possibility that an approximate symmetry of  $\tilde{U}(12)_{SF} \supset \tilde{U}(4)_{D.S.} \otimes U(3)_F$  is realized in nature in the world of hadron spectroscopy.

Here the  $\tilde{U}(12)_{SF}$  symmetry is mathematically the same as the one<sup>4)</sup> that appeared in 1965 to generalize covariantly the static  $SU(6)_{SF}$  symmetry<sup>5)</sup> ( $SU(6)_{SF} \supset SU(2)_S \otimes U(3)_F$ ) in NRQM. However, in the case of  $\tilde{U}(12)_{SF}$  at that time *only the boosted Pauli-spinors*, which reduce to the Pauli-spinors at the hadron rest frame,) are taken as *physical components* out of the fundamental representations of  $\tilde{U}(4)_{D.S.}$ .

Now in the present scheme<sup>\*)</sup> of the  $\tilde{U}(12)$  symmetry *all general Dirac spinors* are, inside of hadrons, to be treated as *physical*.

Accordingly in the covariant classification scheme there exist<sup>\*\*)</sup> the chiral states / chiralons (, which never exist in the conventional scheme based on NRQM), in addition to the Pauli-states / Pauli particles. The purpose of this talk is to review the essential points of the covariant classification scheme and to explain the physical background for the existence of chiral states.

(*Lorentz covariance for local particle*) For this purpose it is instructive to remember the meaning of Lorentz-covariance in the case of local fields.

For the infinitesimal Lorentz transformation  $\Lambda_{\mu\nu}$  of the space time coordinate  $X$  the wave function  $\Phi(X)$  of the particle field transforms by the  $S(\Lambda)$  as,

$$X'_\mu = \Lambda_{\mu\nu} X_\nu \equiv (\delta_{\mu\nu} + \epsilon_{\mu\nu}) X_\nu , \quad (1.1)$$

$$\Phi(X) \rightarrow \Phi'(X') = S(\Lambda)\Phi(X) , \quad (1.2)$$

$$S(\Lambda) \equiv (1 + \frac{i}{2}\epsilon_{\mu\nu}\Sigma_{\mu\nu}) . \quad (1.3)$$

The generators for rotation  $J_i$  and for boost  $K_i$  are defined in terms of  $\Sigma_{\mu\nu}$ , respectively, as

$$J_i \equiv \frac{1}{2}\epsilon_{ijk}\Sigma_{jk} , \quad K_i \equiv i\Sigma_{i4} . \quad (1.4)$$

In the case of Dirac spinors with spin  $J = 1/2$ , the transformation operators/generators for rotation and boost are given, respectively, as

$$S_R(\theta_i) = e^{-i\boldsymbol{\theta}\cdot\mathbf{J}} , \quad J_i \equiv \frac{1}{2}\sigma_i \otimes \rho_0 ; \quad (1.5)$$

$$S_B(\mathbf{P}) = e^{-i\mathbf{b}\cdot\mathbf{K}} , \quad (\mathbf{b} \equiv \hat{\mathbf{v}} \cosh^{-1} v_0), \quad K_i \equiv \frac{i}{2}\rho_1 \otimes \sigma_i ; \quad (1.6)$$

where  $\theta_i$  are rotation angles around the  $i$ -axes,  $v_\mu \equiv P_\mu/M$  is the four-velocity of relevant particles, and  $\rho_i$  and  $\sigma_i$  is the  $2 \times 2$  Pauli-matrices representing the  $4 \times 4$  Dirac matrices as  $\gamma \equiv \rho \otimes \sigma$ . The  $\rho$  and  $\sigma$  concern, respectively, with the transformation between the upper-two components of Dirac-spinor  $\psi$  (as a whole-entitiy) and the lower-two, and with the transformation within the respective two-components as,

$$\psi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \chi_1 \\ \chi_2 \end{pmatrix} \begin{array}{c} \swarrow \sigma \\ \searrow \\ \swarrow \sigma \\ \searrow \end{array} \begin{array}{c} \swarrow \\ \searrow \\ \swarrow \\ \searrow \end{array} \rho . \quad (1.7)$$

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<sup>\*)</sup> We use the term  $\tilde{U}(4)_{\text{D.S.}}$  ( $\tilde{U}(12)_{\text{SF}}$ ) as what implying  $U(4)_{\text{stat.}}$  ( $U(12)_{\text{stat.}}$ ), that is, 4(12)-dimensional unitary group at the rest-frame.

<sup>\*\*)</sup>  Shortly after the symposium, the existence of new narrow resonances,  $D_{s,J}(2317)$  and  $D_{s,J}(2463)$  in  $(c\bar{s})$  system, aroused strong interests among us. They are naturally assigned<sup>19)</sup> as the ground state chiralons with  $J^P = 0^+$  and  $1^+$ , which are chiral partners of  $D_s$  and  $D_s^*$ , respectively.

By inspecting the above formulas, noting that the  $\rho$  is contained only in the booster  $K_i$ , we see that the boosting transformation  $S_B$  plays its role in the moving frame, while becomes identity in the rest-frame of relevant particle as  $S_B(\mathbf{P} \rightarrow 0) \rightarrow 1$ . This means physically that, all the four components are necessary for its covariant description, although only the two-components are enough for description of its spin 1/2 character.

## §2. Covariant framework for describing composite hadrons

(*Attributes and wave function of hadrons*) Our relevant composite hadrons have, as their indispensable attributes, definite mass and spin, definite Lorentz-transformation property, and definite quark-composite structures. Accordingly, in any covariant framework, ①the hadron wave function (WF) must satisfy the Klein-Gordon equation, ②the explicit form of generators for its Lorentz-transformation is to be given, and ③the WF has the spinor-flavor indices  $A(\alpha, a)$  ( $\alpha = 1 \sim 4$  ( $a$ ) denoting Dirac (flavor) index). Thus, we set up the WF for mesons and baryons, respectively, as

$$\begin{aligned} \text{Meson} : \Phi_A^B(x, y) &\sim \psi_A(x) \bar{\psi}^B(y) , \\ \text{Baryon} : \Phi_{A_1 A_2 A_3} &\sim \psi_{A_1}(x_1) \psi_{A_2}(x_2) \psi_{A_3}(x_3) , \end{aligned} \quad (2.1)$$

and assume that they are tensors in the  $\tilde{U}(12)_{SF} \otimes O(3, 1)$  space (the  $O(3, 1)$  being the Lorentz space for the space-time coordinates of constituent quarks).

(*Klein-Gordon equation and mass term*) We start from the Yukawa-type Klein Gordon equation as a basic wave equation<sup>6)</sup>.

$$[\partial^2/\partial X_\mu^2 - \mathcal{M}^2(r_\mu, \partial/\partial r_\mu; \partial/i\partial X_\mu)]\Phi(X, r, \dots) = 0 , \quad (2.2)$$

where  $X_\mu$  ( $r$  for mesons,  $r_1, r_2$  for baryons) are the center of mass (relative) coordinates of hadron systems. It is here to be noted that the Klein-Gordon equation corresponds to Einstein Relation. Here the Klein-Gordon operator, both the  $\partial^2/\partial X_\mu^2$  and the squared mass operator  $\mathcal{M}^2$  (consisting of the part of kinetic motion on relative space-time freedom and of the confining-force part  $\mathcal{M}_{\text{conf}}^2$ ) is assumed to be Lorentz-scalar and  $A, (B)$ -independent, leading in the case of light-quark hadrons to the squared mass spectra with the  $\tilde{U}(12)$  symmetry and also with the chiral symmetry. As its concrete model we apply the covariant oscillator in COQM, leading to the straightly-rising Regge trajectories. The effects due to perturbative QCD and other possible effects are neglected in the symmetric limit. The total WF are separated into the positive (negative)-frequency parts concerning the CM plane-wave motion and expanded in terms of eigen-states of the squared-mass operator,  $\psi_N^{(\pm)}$  satisfying  $\mathcal{M}^2 \psi_N^{(\pm)} = M_N^2 \psi_N^{(\pm)}$ , as

$$\Phi(X, r, \dots) = \sum_N \sum_{\mathbf{P}_N} \left[ e^{iP_N \cdot X} \psi_N^{(+)}(P_N, r, \dots) + e^{-iP_N \cdot X} \psi_N^{(-)}(P_N, r, \dots) \right] . \quad (2.3)$$

(*Expansion of WF on bases of BW-spinor  $\otimes$  Oscillator*) In order to reproduce the success of the conventional NR-classification scheme with approximate LS-symmetry, we will describe the internal WF of relativistic composite hadrons with

a definite mass and a definite total spin ( $J = L + S$ ), which are tensors in the  $\tilde{U}(4)_{\text{D.S.}} \times O(3, 1)$  space, by expanding them in terms of manifestly covariant bases of complete set, being a direct product of eigen-functions in the respective sub-space.

$$\psi_{J,\alpha\cdots}^{\beta\cdots}(P_N, r, \cdots) = \sum_{i,j} c_{ij}^J W_\alpha^{(i)\beta}(P_N) O^{(j)}(P_N, r \cdots) . \quad (2.4)$$

We choose the BW spinors and the covariant oscillator functions (with a definite metric type) as the respective bases.

(*Spinor WF*) The internal WF is, concerning the spinor freedom, expanded in terms of complete set of multi-Dirac spinors, Bargmann-Wigner (BW) spinors. The BW spinors are defined as solutions of the relevant local Klein-Gordon equation:

$$(\partial^2/\partial X_\mu^2 - M^2)W_{\alpha\cdots}^{\beta\cdots}(X) = 0 \quad (2.5)$$

$$W_{\alpha\cdots}^{\beta\cdots}(X) \equiv \sum_{\mathbf{P}} (e^{iPX} W_{\alpha\cdots}^{(+)\beta\cdots}(P) + e^{-iPX} W_{\alpha\cdots}^{(-)\beta\cdots}(P)). \quad (2.6)$$

(*Space-time WF*) The internal WF is, concerning the relative space-time freedom, expanded in terms of the complete set of covariant oscillator eigen-functions; where, by applying a Lorentz-invariant subsidiary condition<sup>7)</sup> to “freeze” the relative-time freedom,

$$\langle P_\mu r_\mu \rangle = \langle P_\mu p_\mu \rangle = 0 \xrightarrow{\mathbf{P}=0} O(3, 1) \approx O(3) . \quad (2.7)$$

The original symmetry  $O(3, 1)$  is reduced into the non-relativistic  $O(3)$  symmetry.

Here it is noteworthy that the above choice of bases is desirable from the phenomenological facts i) that the constituent quark inside of hadrons behaves like a free Dirac-particle<sup>\*)</sup> (implied by BW-spinors) and ii) that in the global structure of hadron spectra (Regge trajectory and so on) is well described by the corresponding oscillator potential.

(*Spinor WF and chiral states*) In order to make clear the physical background for the chiral states we describe the spinor WF of the ground states of mesons and baryons, neglecting the internal space-time variables. First we define the Dirac spinor for quarks  $W_q$  and its Pauli-conjugate for anti-quarks  $\bar{W}_{\bar{q}}$  as BW spinors with single index, respectively, by

$$\begin{aligned} \text{Dirac spinor } \psi_{q,\alpha}(X) &= \sum_{\mathbf{P},r} [e^{iPX} W_{q,\alpha}^{(+)}(P) + e^{-iPX} W_{q,\alpha}^{(-)}(P)] \\ W_{q,\alpha}^{(+)}(P) &= u_\alpha(P), \quad W_{q,\alpha}^{(-)}(P) = u_\alpha(-P). \end{aligned} \quad (2.8)$$

$$\begin{aligned} \bar{\psi}_{\bar{q},\alpha}(X) &= \sum_{\mathbf{P},s} [e^{iPX} \bar{W}_{\bar{q},\alpha}^{(+)}(P) + e^{-iPX} \bar{W}_{\bar{q},\alpha}^{(-)}(P)] \\ \bar{W}_{\bar{q},\alpha}^{(+)}(P) &= \bar{v}_{\bar{q},\alpha}(P), \quad \bar{W}_{\bar{q},\alpha}^{(-)}(P) = \bar{v}_{\bar{q},\alpha}(-P). \end{aligned} \quad (2.9)$$

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<sup>\*)</sup> The BW-spinors with total hadron momentum  $P_\mu$  and  $M$  are easily shown to be equivalent to the product of free Dirac spinors of the respective constituent “exciton-quarks” with momentum  $p_{N,\mu}^{(i)} \equiv \kappa^{(i)} P_{N,\mu}$  and mass  $m_N^{(i)} \equiv \kappa^{(i)} M_N$  ( $\sum_i \kappa^{(i)} = 1$ ). See the reference<sup>3)</sup>.

They take the following form at the hadron rest frame as

$$W(\mathbf{P} = \mathbf{0}); \quad W_{q,r}^{(+)} = \begin{pmatrix} \chi_r \\ 0 \end{pmatrix} \quad \rho_3 = + \quad , \quad W_{q,r}^{(-)} = \begin{pmatrix} 0 \\ \chi_r \end{pmatrix} \quad \rho_3 = - \quad (2.10)$$

$$\bar{W}_{\bar{q},s}^{(+)} = (0, \bar{\chi}_s) \quad \bar{\rho}_3 = + \quad , \quad \bar{W}_{\bar{q},s}^{(-)} = (\bar{\chi}_s, 0) \quad \bar{\rho}_3 = -$$

It is to be noted that all Dirac spinors with positive and negative values of  $\rho_3(\bar{\rho}_3)$  spin for quarks (anti-quarks) are required as members of complete set of expansion bases inside of hadrons. The spinor WF for ground state mesons are given by bi-Dirac spinors as BW spinors with two indices as

$$\text{Meson spinor} \quad W_\alpha^\beta(P) = W_q(P)_\alpha \bar{W}_{\bar{q}}(P)^\beta$$

$$W_\alpha^\beta(\mathbf{P} = \mathbf{0}); (\rho_3, \bar{\rho}_3) = (+, +) : \text{boosted Pauli states} \quad (2.11)$$

$$(+, -), (-, +), (-, -) : \text{“Chiral States”}.$$

The spinor WF for ground states of baryons and anti-baryons are given similarly by tri-Dirac spinors as BW spinors with three indices as

$$\text{Baryon spinor} \quad W_{\alpha\beta\gamma}^{(B)}(P) = W_{q,\alpha}(P) W_{q,\beta}(P) W_{q,\gamma}(P)$$

$$W_{\alpha\beta\gamma}(\mathbf{P} = \mathbf{0}); (\rho_3^{(1)}, \rho_3^{(2)}, \rho_3^{(3)}) = (+, +, +) : \text{boosted Pauli states} \quad (2.12)$$

$$(+, +, -), (+, -, -) : \text{“Chiral States”}$$

$$W_{\alpha\beta\gamma}^{(\bar{B})}(P) = W^{(B)}\{ W_{q,\alpha} \rightarrow W_{\bar{q},\alpha} \} . \quad (2.13)$$

The meson WF with  $(\rho_3, \bar{\rho}_3) = (+, +)$  and the baryon WF with  $(\rho_3^{(1)}, \rho_3^{(2)}, \rho_3^{(3)}) = (+, +, +)$  are the multi-boosted Pauli spinors which reduce to the multi-NR Pauli spinors at the rest frame, while the meson and baryon WF with the other values of  $(\rho_3, \bar{\rho}_3)$  and of  $(\rho_3^{(1)}, \rho_3^{(2)}, \rho_3^{(3)})$ , respectively, describe the chiral states of mesons and baryons, which newly appear in the covariant classification scheme.

Here it is to be noted that the intrinsic spin-freedom for totality of BW spinors of  $q\bar{q}$  mesons is  $4 \times 4 = 16$ , four times of  $2 \times 2 = 4$  for boosted Pauli spinors. For  $qqq$  baryons  $G(P)$  and  $F(P)$ , which include one and two negative  $\rho_3$  Dirac spinors, respectively, appear in addition to the conventional  $E(P)$  (with all positive  $\rho_3$  Dirac spinors), boosted multi-Pauli spinor. It is also to be noted that, although the BW equation with a definite mass itself is not  $\tilde{U}(4)_{DS}$  symmetric, the Klein-Gordon equation with a definite mass-squared is generally  $\tilde{U}(4)_{DS}$  symmetric.

(Covariant quark representation of hadrons and rule for chiral transformation)  
Any transformation rule for composite hadrons are able to be derived from that for constituent quarks in the covariant classification scheme. An interesting and useful example is that for chiral transformation:

$$(\text{Chiral Transf.}) \quad \psi_q \rightarrow \psi'_q = e^{i\beta\gamma_5} \psi_q , \quad (2.14)$$

$$\text{Baryon; } W_{\alpha_1\alpha_2\alpha_3} \rightarrow W'_{\alpha_1\alpha_2\alpha_3} = \left[ \Pi_{i=1}^3 (e^{i\beta\gamma_5^{(i)}}) W \right]_{\alpha_1\alpha_2\alpha_3} ,$$

$$\text{Meson}; W_\alpha^\beta \rightarrow W_\alpha'^\beta = \left[ e^{i\beta\gamma_5} W e^{i\beta\gamma_5} \right]_\alpha^\beta. \quad (2.15)$$

In order to see the physical meaning of the chiral transformation of hadrons we note the results of operating the generators of chiral transformation on the constituent quarks inside of hadrons.

$$\begin{aligned} u_\pm(P) &\rightarrow u'_{(\pm)}(P) = -\gamma_5 u(P)_{(\pm)} = u(-P)_{(\mp)}, \\ v_\pm(P) &\rightarrow v'_{(\pm)}(P) = -\gamma_5 v(P)_{(\pm)} = v(-P)_{(\mp)}. \end{aligned} \quad (2.16)$$

Noting that  $P_\mu \equiv (\mathbf{P}, iE_{\mathbf{P}} = i\sqrt{\mathbf{P}^2 + M^2})$  the transformation changes the positive  $\rho_3$  ( $\bar{\rho}_3$ )-quark (anti-quark) spinors into the negative ones, and vice versa. This implies that the chiral transformation transforms the members of complete set of BW-spinors for each other. Accordingly, if  $\mathcal{M}^2$  operator is a Lorentz-scalar and independent of Dirac indices, the hadron squared-mass spectra have effectively the chiral symmetry, in addition to the  $\tilde{U}(4)_{\text{D.S.}}$  symmetry and  $\tilde{U}(12)_{\text{SF}}$  symmetry(, including the flavor freedom in the case of light quark system). Here it is to be stressed that the intention in this work is not to treat a dynamical problem from a conventional composite picture, but is to propose a kinematical framework for describing composite hadrons covariantly. The validity of the above assumption is checked only by comparing its predictions with experimental and phenomenological facts.

### §3. Level Structure of Mesons and Baryons

(*Symmetry of ground states*) In our scheme hadrons are generally classified as the members of multiplet in the  $\tilde{U}_{SF}(12) \times O(3,1)$  scheme. The light-quark ground state mesons and baryons are assigned to the representations  $(\mathbf{12} \times \mathbf{12}^*) = \mathbf{144}$  and  $(\mathbf{12} \times \mathbf{12} \times \mathbf{12})_{\text{Symm}} = \mathbf{364}$  of the  $\tilde{U}(12)_{SF}$  symmetry, respectively. (See, Tables I and II.) The numbers of freedom of spin-flavor WF in NRQM are  $\mathbf{6} \times \mathbf{6}^* = \mathbf{36}$  for mesons and  $(\mathbf{6} \times \mathbf{6} \times \mathbf{6})_{\text{Symm.}} = \mathbf{56}$  for baryons (and  $\mathbf{56}^*$  for antibaryons): These numbers in COQM are extended to  $\mathbf{144}$  for mesons, and  $\mathbf{364} = \mathbf{182}$  (for baryons) +  $\mathbf{182}$  (for anti-baryons) for baryon-and-antibaryons, respectively.

Mesons:	$(\mathbf{12} \times \mathbf{12})$	$= \mathbf{144}$	<b>Chiral States</b>					
	$P_s^{(N)}$	$V_\mu^{(N)}$	$Ps^{(E)}$	$V_\mu^{(E)}$	$S^{(N)}$	$A_\mu^{(N)}$	$S^{(E)}$	$A_\mu^{(N)}$
$J^{PC}$	$0^{-+}$	$1^{--}$	$0^{-+}$	$1^{--}$	$0^{++}$	$1^{++}$	$0^{+-}$	$1^{+-}$

Table I. Quantum numbers of ground-state meson multiplet in  $\tilde{U}(12)_{SF}$  symmetry

Baryons:	$(\mathbf{12} \times \mathbf{12} \times \mathbf{12})_{\text{Sym}} = \mathbf{364} = \mathbf{182}_B + \mathbf{182}_{\bar{B}}$				
	$\mathbf{182}$	$\mathbf{56}$	$\Delta_{3/2}^\oplus$	$N_{1/2}^\oplus$	
		$\mathbf{70}$	$\Delta_{1/2}^{\ominus(\oplus)}$	$N_{3/2}^{\ominus(\oplus)}$	$N_{1/2}^{\ominus(\oplus)}$
		$\mathbf{56}'$	$\Delta_{3/2}^{\oplus(\ominus)}$	$N_{1/2}^{\oplus(\ominus)}$	
		<b>Chiral States</b>			

Table II. Quantum numbers of ground-state baryon multiplet in  $\tilde{U}(12)_{SF}$  symmetry

Inclusion of heavy quarks<sup>\*)</sup> is straightforward: The WF of general  $q$  and/or  $Q$  hadrons become tensors in  $O(3,1) \otimes [\tilde{U}(4)_{\text{D.S.}} \otimes SU(3)_F]_q \otimes [\tilde{U}(4)_{\text{P.S.}} \otimes U(1)_F]_Q$ .

(*Level structure of ground state mesons*)<sup>3)</sup> In Table III we have summarized the properties of ground state mesons in the light and/or heavy quark systems. It is remarkable that there appear new multiplets of the scalar and axial-vector mesons, chiralons, in the  $q\bar{Q}$  and  $Q\bar{q}$  systems and that in the  $q\bar{q}$  systems the two sets (Normal and Extra) of pseudo-scalar and of vector meson nonets exist. The  $\pi$  nonet ( $\rho$  nonet) is assigned to the  $P_s^{(N)}$  ( $V_\mu^{(N)}$ ) state.

	mass	Approx. Symm.	Spin WF	$SU(3)$	Meson Type
$Q\bar{Q}$	$m_Q + m_{\bar{Q}}$	$LS$ symm.	$u_Q(P)\bar{v}^Q(P)$	$\underline{1}$	$P_s, V_\mu$
$q\bar{Q}$	$m_q + m_{\bar{Q}}$	$q$ -Chiral Symm.	$u_q(P)\bar{v}^Q(P)$	$\underline{3}$	$P_s, V_\mu$
		$\bar{Q}$ -Heavy Q. Symm.	$u_q(-P)\bar{v}^Q(P)$	$\underline{3}$	$S, A_\mu$
$Q\bar{q}$	$m_Q + m_{\bar{q}}$	$\bar{q}$ -Chiral Symm.	$u_Q(P)\bar{v}^q(P)$	$\underline{3}^*$	$P_s, V_\mu$
		$Q$ -Heavy Q. Symm.	$u_Q(P)\bar{v}^q(-P)$	$\underline{3}^*$	$S, A_\mu$
$q\bar{q}$	$m_q + m_{\bar{q}}$	Chiral Symm.	$\frac{1}{\sqrt{2}}(u(P)\bar{v}(P) \pm u(-P)\bar{v}(-P))$	$\underline{9}$	$P_s^{(N,E)}, V_\mu^{(N,E)}$
			$\frac{1}{\sqrt{2}}(u(P)\bar{v}(-P) \pm u(-P)\bar{v}(P))$	$\underline{9}$	$S^{(N,E)}, A_\mu^{(N,E)}$

Table III. Level structure and the spinor wave function of ground-state mesons

(*Level structure of mesons in general*)<sup>3)</sup> The global mass spectra of the ground and excited state mesons are given by

$$M_N^2 = M_0^2 + N\Omega = (m_N^{(1)} + m_N^{(2)})^2. \quad (3.1)$$

Their quantum numbers are given in Table IV. Here it is to be noted that some chiralons have the “exotic” quantum numbers from the conventional NRQM viewpoint.

$(q\bar{q})$	$P$	$C$	$N$	$(q\bar{Q} \text{ or } Q\bar{q})$	$P$	$N$
$\{P_s^{(N)}, V_\mu^{(N)}\} \otimes \{L, N\}$	$(-1)^{L+1}$	$(-1)^{L+S}$	all	$\{P_s, V_\mu\} \otimes \{L, N\}$	$(-1)^{L+1}$	all
$\{P_s^{(E)}, V_\mu^{(E)}\} \otimes \{L, N\}$	$(-1)^{L+1}$	$(-1)^{L+S}$	0, 1	$\{S, A_\mu\} \otimes \{L, N\}$	$(-1)^L$	0, 1
$\{S^{(N)}, A_\mu^{(N)}\} \otimes \{L, N\}$	$(-1)^L$	$(-1)^L$	0, 1	$(Q\bar{Q})$	$P$	$N$
$\{S^{(E)}, A_\mu^{(E)}\} \otimes \{L, N\}$	$(-1)^L$	$(-1)^{L+1}$	0, 1	$\{P_s, V_\mu\} \otimes \{L, N\}$	$(-1)^{L+1}$	all

Table IV. Level structure of Mesons: We are able to infer<sup>8)</sup> that the chiral symmetry concerning the light quarks is valid (still effective) for the ground (first excited) state of  $n\bar{n}$  and  $n\bar{Q}$  meson systems, while the symmetry will prove invalid from the  $N$ -th ( $N \geq 2$ ) excited hadrons.

(*Level structure of baryons*) The baryon WF in Eq. (2.12) should be full-symmetric (except for the color freedom) under exchange of constituent quarks: The full-symmetric total WF in the extended scheme is obtained, in the three ways, as a product of the sub-space  $\rho$ ,  $\sigma$  and  $F$  WF with respective symmetric properties. (As for details, see ref. 9).)

Here it is remarkable that there appear chiralons in the ground states. That is, the extra positive parity  $\underline{56}'$ -multiplet of the static  $SU(6)$  and the extra negative

<sup>\*)</sup> Concerning the heavy quarks, we take only the boosted-Pauli spinors,  $\tilde{U}(4)_{P.S.}$  as physical ones out of total members of  $\tilde{U}(4)_{D.S.}$ , because that for them the chiral symmetry is not valid. The  $\tilde{U}(4)_{P.S.}$  symmetry becomes a two dimensional unitary symmetry at the rest frame  $U(2)_{\text{stat}}$ .

parity **70**-multiplet of the  $SU(6)$  in the low mass region. It is also to be noted that the chiralons in the first excited states are expected to exist. The above consideration on the light-quark baryons are extended directly to the general light and/or heavy quark baryon systems: The chiralons are expected to exist also in the  $qqQ$  and  $qQQ$ -baryons, while no chiralons in the  $QQQ$  system.

#### §4. Remarks on Interaction among Hadrons

(*Chiral symmetric spectator*) In treating interactions between hadrons resorting on their quark-composite structure, the effective interaction vertices are generally written as a product of the two parts, the one concerning the fundamental quark-interactions and the other concerning the overlap of spectator quarks. The overlapping spectator-interaction, which is considered to be due to QCD, should be chiral symmetric in the ideal limit of neglecting the effects of spontaneous breaking. However, the bilinear scalar-covariant between Dirac spinor  $\psi$  and their Pauli-conjugate  $\bar{\psi} \equiv \psi^\dagger \gamma_4$  ( $\bar{\psi} \equiv 1 \psi$ ) violates maximally the chiral symmetry, leading to the non-chiral symmetric effective hadron interaction, even in the case of symmetric fundamental quark interactions. Correspondingly we define the “unitary WF” (its conjugate) of mesons and of baryons, respectively, (revising the WF (2.1)) by

$$\text{Meson : } \Phi_{U,A}^B \equiv (\Phi \bar{\gamma}_4)_A^B \sim \psi_A \psi^{\dagger B}, \quad \bar{\Phi}_{U,B}^A \equiv (\bar{\Phi} \bar{\gamma}_4)_B^A \sim \psi_B \psi^{\dagger A} \quad (4.1)$$

$$(\bar{\Phi} \equiv \gamma_4 \Phi^\dagger \gamma_4, \quad \bar{\gamma}_4 = -i v \cdot \gamma, \quad v_\mu \equiv P_\mu/M) .$$

$$\text{Baryon : } \Phi_{U,A_1 A_2 A_3} \equiv \Phi_{A_1 A_2 A_3} \sim \psi_{A_1} \psi_{A_2} \psi_{A_3} ,$$

$$\bar{\Phi}_{U}^{B_1 B_2 B_3} \equiv (\bar{\Phi} \Pi_{i=1}^3 \gamma_4^{(i)})^{B_1 B_2 B_3} \sim \psi^{\dagger B_1} \psi^{\dagger B_2} \psi^{\dagger B_3} \quad (4.2)$$

$$(\bar{\Phi} \equiv \Phi^\dagger \Pi_{i=1}^3 \gamma_4^{(i)}, \quad \bar{\gamma}_4^{(i)} = -i v \cdot \gamma^{(i)}) .$$

These new WF are so defined as leading to their bilinear scalar-covariant being chiral symmetric and also equal to the unitary overlap in the static limit

$$\langle \bar{\Phi}_U \Phi_U \rangle \xrightarrow{\mathbf{v} \rightarrow \mathbf{0}} \langle \Phi^\dagger \Phi \rangle . \quad (4.3)$$

Here it is to be noted that each spectator overlap-interaction now leads to an interesting selection rule ( $\rho_3$ -line rule). In the static limit ( $\mathbf{v} \rightarrow \mathbf{0}$ )  $\rho_3$  value conserves along each spectator-quark line.

(*Chiral-symmetric effective EM-currents*) The electromagnetic (EM) effective hadron interactions should be chiral symmetric in the ideal limit. We are able to derive the effective EM hadron currents (which are conserved and chiral symmetric), following directly to the method of minimal substitution<sup>10)</sup> in the framework of COQM (with replacement of  $\Phi$  by  $\Phi_U$ ). That is, in the multi-local Lagrangian density of action  $S_I^{EM}$ , leading to our basic Klein-Gordon equation (2.2), the derivative on quark coordinates is replaced by the gauge-covariant one. The results are given as follows:

$$S_I^{EM} = \int \Pi_{i=1}^3 d^4 x_i \sum_{i=1}^3 j_{i,\mu}^{EM}(x_1, \dots) A_\mu(x_i) \equiv \int d^4 X J_\mu^{EM}(X) A_\mu(X),$$



$$j_{i,\mu}^{EM}(x_1, \dots) \propto e_i \langle \bar{\Phi}_U(x_1, \dots) \overleftrightarrow{\partial}_{i,\mu} \Phi_U(x_1, \dots) \rangle, \quad (4.4)$$

where only the case of baryons is shown as an example. In the actual application this current should be revised by taking into account the symmetry breaking effects above mentioned.

## §5. Candidates of Chiral States and Concluding Remarks

(*Experimental candidates of chiral particles*) In our level-classification scheme a series of new type of multiplets, chiralons, are predicted to exist in the ground and the first excited states. Presently we can give only a few experimental candidates or indications for them:

( *$q\bar{q}$ -mesons*)\*) One of the most important candidates is the scalar  $\sigma$  nonet to be assigned as  $S^{(N)}(1S_0) : [\sigma(600), \kappa(900), a_0(980), f_0(980)]$ . The existence of  $\sigma(600)$  seems to be established<sup>1)</sup> through the analyses of, especially,  $\pi\pi$ -production processes. Some evidences for  $\kappa(800-900)$  in  $K\pi$  scattering phase shift<sup>11)</sup> had been given formerly. Its firm experimental evidences in the production process<sup>12)</sup> and through the decay process<sup>13)</sup> were reported recently.

In our scheme respective two sets of  $P_s$ - and of  $V_\mu$ -nonets, to be assigned as  $P_s^{(N,E)}(1S_0)$  and  $V_\mu^{(N,E)}(3S_1)$ , are to exist: The vector mesons<sup>14)</sup>  $\rho(1250)$  and  $\omega(1250)$ , suspected to exist for long time, are naturally able to be assigned as the members of  $V_\mu^{(E)}(3S_1)$ -nonet;

Out of the three established  $\eta$ , [ $\eta(1295)$ ,  $\eta(1420)$ ,  $\eta(1460)$ ] at least one extra, plausibly  $\eta(1295)$  with the lowest mass, may belong to  $P_s^{(E)}(1S_0)$  nonet.

Recently the existence of two “exotic” particles  $\pi_1(1400)$  and  $\pi_1(1600)$  with  $J^{PC} = 1^{-+}$  and  $I = 1$ , observed<sup>15)</sup> in the  $\pi\eta$ ,  $\rho\pi$  and other channels, is attracting strong interests among us. These exotic particles with a mass around 1.5GeV may be naturally assigned as the first excited states  $S^{(E)}(1P_1)$  and  $A_\mu^{(E)}(3P_1)$  of the chiralons.

( *$q\bar{Q}$  or  $Q\bar{q}$ -mesons*\*\*) Recently we have shown some indications for existence of the following two chiralons<sup>16)</sup> in  $D$ - and  $B$ -meson systems obtained through analyses of the  $\Upsilon(4S)$  or  $Z^0$  decay processes, respectively, as  $D_1^\chi(2310)$  with  $J^P = 1^+$  in  $D_1^\chi \rightarrow D^* + \pi$  and  $B_0^\chi(5520)$  with  $J^P = 0^+$  in  $B_0^\chi \rightarrow B + \pi$ .

( *$qqq$ -baryons*) The two facts have been a longstanding problem that the Roper resonance  $N(1440)_{1/2^+}$  is too light to be assigned as radial excitation of  $N(939)$  and that  $\Lambda(1405)_{1/2^-}$  is too light as the  $L = 1$  excited state of  $\Lambda(1116)$ . In our new scheme these two problems dissolve<sup>17)</sup> in principle, because in the ground states there exist the two **56** of  $SU(6)$  with positive parity,  $E(\mathbf{56}^+)$  and  $F(\mathbf{56}^+)$ , and one **70** of  $SU(6)$  with negative parity,  $G(\mathbf{70}^-)$ .

(*Concluding remarks*) We have summarized in this talk the essential points of the covariant level-classification scheme, which has, we believe, a possibility to solve the serious problem in hadron spectroscopy mentioned in §1. In this connection

\*) A tentative assignment in the covariant classification scheme of all the light-quark mesons with mass  $\lesssim 1.8\text{GeV}$ , reported in PDG, was made rather satisfactorily. (See Ref. 18).)

\*\*) See also the footnote on p.2 .

further investigations, both experimental and theoretical, for chiral states predicted in this scheme, are urgently required for new development of hadron physics.

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